

$|V_{us}|$ from hadronic τ decays¹

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Abstract

We review the status and future prospects of the determination of the CKM matrix element $|V_{us}|$ using inclusive strange hadronic τ decay data. We also review the results for $|V_{us}|$ extracted from experimental measurements of some exclusive τ decay channels such as $\tau \rightarrow K\nu$ and $\tau \rightarrow \pi\nu$.

1 Introduction

The hadronic decays of the τ lepton serve as an ideal system to study low-energy QCD under rather clean conditions [1]. The study of these processes has allowed the determination of the strong coupling α_s to a level of precision only achieved by lattice determinations: $\alpha_s(m_\tau) = 0.334 \pm 0.014$ [2]. The fact that the strong coupling calculated as such a low scale, m_τ agrees with direct measurements of $\alpha_s(M_Z)$ at the Z peak when run to $\mu = M_Z$, provides the most precise test of asymptotic freedom [3]

$$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0014 \pm 0.0016_\tau \pm 0.0027_Z. \quad (1)$$

This precise determination of α_s has been possible thanks to the detailed investigation, both experimentally (by ALEPH and OPAL at LEP, CLEO at CESR, and the B factories BaBar and Belle) and theoretically [4], of the hadronic decay rate of the τ lepton

$$R_\tau \equiv \frac{\Gamma[\tau^- \rightarrow \text{hadrons}(\gamma)]}{\Gamma[\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau(\gamma)]} = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}. \quad (2)$$

The experimental separation of the Cabibbo-allowed decays ($R_{\tau,V} + R_{\tau,A}$) and Cabibbo-suppressed modes ($R_{\tau,S}$) into strange particles allows us to also study SU(3)-breaking effects through the difference

$$\delta R_\tau \equiv \frac{R_{\tau,V+A}}{|V_{ud}|^2} - \frac{R_{\tau,s}}{|V_{us}|^2}. \quad (3)$$

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Flavour independent uncertainties drop out in the difference in Eq. (3), which is dominated by the strange quark mass, providing an ideal place to determine m_s [5]. This determination, however, is very sensitive to the value of $|V_{us}|$ employed. It appears thus natural to turn things around and to determine $|V_{us}|$ with an input for m_s using Eq. (3) [6]

$$|V_{us}|^2 = \frac{R_{\tau,S}^{exp}}{\frac{R_{\tau,V+A}^{exp}}{|V_{ud}|^2} - \delta R_{\tau}^{theor}}. \quad (4)$$

The main advantage of extracting $|V_{us}|$ from hadronic τ decays via (4) is that δR_{τ}^{theor} is around an order of magnitude smaller than the ratio $R_{\tau,V+A}^{exp}/|V_{ud}|^2$, and thus the determination of $|V_{us}|$ is dominated by quantities that are measured experimentally. The final precision that can be achieved is then an experimental issue.

2 Theoretical calculation of δR_{τ}

The hadronic decay rate of the τ in Eq. (2) is related to the longitudinal (J=0) and transversal (J=1) part of vector and axial-vector two-point correlation functions via

$$R_{\tau} = 12\pi \int_0^{M_{\tau}^2} \frac{ds}{M_{\tau}^2} \left(1 - \frac{s}{M_{\tau}^2}\right)^2 \left[\left(1 + 2\frac{s}{M_{\tau}^2}\right) \text{Im } \Pi^T(s) + \text{Im } \Pi^L(s) \right], \quad (5)$$

where the relevant combination of two-point functions is

$$\Pi^J(s) \equiv |V_{ud}|^2 \left\{ \Pi_{V,ud}^J(s) + \Pi_{A,ud}^J(s) \right\} + |V_{us}|^2 \left\{ \Pi_{V,us}^J(s) + \Pi_{A,us}^J(s) \right\}, \quad (6)$$

with the functions $\Pi_{ij}^J(s)$ given by $\Pi_{V,ij}^{\mu\nu}(q) \equiv i \int d^4x e^{iq \cdot x} \langle 0 | T \left([V_{ij}^{\mu}]^{\dagger}(x) V_{ij}^{\nu}(0) \right) | 0 \rangle$ and $\Pi_{A,ij}^{\mu\nu}(q) \equiv i \int d^4x e^{iq \cdot x} \langle 0 | T \left([A_{ij}^{\mu}]^{\dagger}(x) A_{ij}^{\nu}(0) \right) | 0 \rangle$. The vector and axial-vector currents are defined as $V_{ij}^{\mu} \equiv \bar{q}_i \gamma^{\mu} q_j$ and $A_{ij}^{\mu} \equiv \bar{q}_i \gamma^{\mu} \gamma_5 q_j$.

Using the analytic properties of the correlators, we can rewrite Eq. (5) as a contour integral running counterclockwise around the circle $|s| = m_{\tau}$ in the complex s -plane

$$R_{\tau} = -i\pi \oint_{|s|=M_{\tau}^2} \frac{ds}{s} \left[1 - \frac{s}{M_{\tau}^2} \right]^3 \left\{ 3 \left[1 + \frac{s}{M_{\tau}^2} \right] D^{L+T}(s) + 4D^L(s) \right\}, \quad (7)$$

where the Adler functions D are defined by

$$D^{L+T}(s) \equiv -s \frac{d}{ds} [\Pi^{L+T}(s)]; \quad D^L(s) \equiv \frac{s}{M_{\tau}^2} \frac{d}{ds} [s \Pi^L(s)]. \quad (8)$$

The contributions to $R_{\tau,V+A}$ and $R_{\tau,S}$, which enter in the definition of the SU(3) breaking difference (3), are given by the terms proportional to $|V_{ud}|^2$ and $|V_{us}|^2$ respectively in the decomposition of the correlation functions in Eq. (6).

At large enough euclidean $Q^2 \equiv -s$, the correlation functions $\Pi^{L+T}(Q^2)$ and $\Pi^L(Q^2)$ can be computed using operator product expansion (OPE) techniques as a series of local gauge-invariant operators of increasing dimension, times appropriate inverse powers of s . Performing the complex integration in (7) we can express the SU(3)-breaking difference as

$$\delta R_\tau = \frac{R_{\tau,V+A}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2} = N_c S_{EW} \sum_{D \geq 2} [\delta_{ud}^{(D)} - \delta_{us}^{(D)}], \quad (9)$$

where the symbols $\delta_{ij}^{(D)}$ stand for corrections of dimension D in the OPE which contain implicit suppression factors of $1/m_\tau^D$.

An extensive theoretical analysis of δR_τ was performed in Ref. [7]. The authors included contributions of dimension two (proportional to m_s^2) and four in the OPE, and estimates of dimension six operators using the vacuum saturation approximation. The authors of Ref. [8] advocated for the use of a phenomenological description of the longitudinal component of δR_τ , which avoids the large uncertainty associated with the bad convergence of the corresponding QCD perturbative corrections. Following Ref. [8] but using the prescription in Ref. [9] to treat the perturbative QCD correction to the $L + T$ $D = 2$, namely, averaging over contour improved and fixed order results for the asymptotically summed series and taking half of the difference as the uncertainty associated with the truncation of the series, one gets [9]

$$\delta R_{\tau,th} = (0.1544 \pm 0.0037) + (9.3 \pm 3.4)m_s^2 + (0.0034 \pm 0.0028) = 0.239 \pm 0.030, \quad (10)$$

where m_s is the strange quark mass in the \overline{MS} scheme and at a scale $\mu = 2$ GeV. The first term contains the phenomenological longitudinal contributions, the second term contains the $L + T$ perturbative $D = 2$ contribution, while the last term stands for the rest of the contributions. Notice that contributions of dimension 4 and higher in the OPE are negligible with current uncertainties in the $D = 2$ perturbative series. The final number in Eq. (10) is obtained using the average over lattice determinations of the strange quark mass, $m_s^{\overline{MS}}(2 \text{ GeV}) = 93.4 \pm 1.1$ [10]. The result in Eq. (10) agrees within errors with those obtained by using different prescriptions for the perturbative $L + T$ $D = 2$ series in Refs. [11, 12].

Using the result in (10), together with the most recent averages of experimental results [16], $R_{\tau,V+A} = 3.4671 \pm 0.0084$ and $R_{\tau,S} = 0.1612 \pm 0.0028$, and $|V_{ud}| = 0.97425 \pm 0.0022$ [13], we get

$$|V_{us}| = 0.2173 \pm 0.0020_{\text{exp}} \pm 0.0010_{\text{th}} = 0.2173 \pm 0.0022. \quad (11)$$

A sizeable fraction of the strange branching fraction is due to the decay $\tau \rightarrow K\nu_\tau$, which can be predicted theoretically with smaller errors than the direct experimental measurements [14]. If the experimental measurements for this decay and for

$\tau \rightarrow K^- \pi^0 \nu_\tau$ and $\tau \rightarrow \pi^- \overline{K}^0 \nu_\tau$ are replaced by theoretical predictions using kaon branching fractions [15], $R_{\tau,S}$ increases about 2.5% and the CKM matrix element is shifted up to $|V_{us}| = 0.2203 \pm 0.0025$.

3 V_{us} from exclusive τ decays

Some exclusive τ decay channels can also be used for the extraction of $|V_{us}|$, given the value of non-perturbative parameters such as f_K , f_K/f_π , or the vector form factor at zero momentum transfer $f_+(0)$. Preliminary results for $f_+(0)|V_{us}|$, using the decay rates $\Gamma(\tau \rightarrow K\pi\nu)$ and a parametrization of the relevant form factors based on dispersion relations can be found in [15].

One can also consider the branching ratio

$$B(\tau \rightarrow K\nu) = \frac{G_F^2 f_K^2 |V_{us}|^2 m_\tau^3}{16\pi\hbar} \left(1 - \frac{m_K^2}{m_\tau^2}\right) S_{EW}, \quad (12)$$

and the ratio

$$\frac{B(\tau \rightarrow K\nu)}{B(\tau \rightarrow \pi\nu)} = \frac{f_K^2 |V_{us}|^2 (1 - m_K^2/m_\tau^2)^2}{f_\pi^2 |V_{ud}|^2 (1 - m_\pi^2/m_\tau^2)^2} \frac{r_{LD}(\tau^- \rightarrow K^- \nu_\tau)}{r_{LD}(\tau^- \rightarrow \pi^- \nu_\tau)}, \quad (13)$$

where the r_{LD} 's are long-distance EW radiative correction. The non-perturbative physics is encoded in the kaon decay constant f_K and the ratio of decay constants f_K/f_π respectively, which are calculated with high precision using lattice techniques. Using the most recent experimental averages for $B(\tau \rightarrow K\nu)$ and $B(\tau \rightarrow \pi\nu)$ [16], the values of r_{LD} also from [16], the lattice averages for f_K and f_K/f_π [10], and $|V_{ud}| = 0.97425 \pm 0.0022$ [13], one gets [16]

$$|V_{us}|_{\tau K} = 0.2214 \pm 0.0022, \quad |V_{us}|_{\tau K\pi} = 0.2229 \pm 0.0021 \quad (14)$$

4 Conclusions

The SU(3) breaking difference between Cabibbo-suppressed and Cabibbo-allowed hadronic τ decay data, δR_τ , has the potential to give the most precise determination of $|V_{us}|$. The theoretical error in Eq. (11) is already at the same level of precision as more accurate determinations from K leptonic and semileptonic decays. The final uncertainty becomes thus an experimental issue and will eventually be reduced with the full analysis of the BaBar and Belle data [17], and even further at future facilities such as Belle II.

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